testenzia

JEE Formula Sheet

Complete Mathematics Formula Collection

A comprehensive compilation of essential mathematics formulas for JEE Main and Advanced. Curated by the testenzia team for maximum exam effectiveness.

Table of Contents

Trigonometry & Inverse Functions

Sum and Difference Formulas

SINE ADDITION

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

> *Product form:* $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

COSINE ADDITION

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

> $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

TANGENT ADDITION

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

COTANGENT

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

Derived Identities

$$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A + B) \sin(A - B)$$

$$\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A + B) \cos(A - B)$$

Product-to-Sum & Sum-to-Product

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

Multiple Angle Formulas

DOUBLE ANGLE

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$

> *Half-angle:* $2\cos^2\frac{\theta}{2} = 1+\cos\theta$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

TRIPLE ANGLE

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

Range of Trigonometric Expression

$$-\sqrt{a^2+b^2} \leq a\sin\theta + b\cos\theta \leq \sqrt{a^2+b^2}$$

Sine and Cosine Series

$$\sin\theta + \sin(\theta+\alpha) + \cdots + \sin(\theta+(n-1)\alpha) = \frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\sin\left(\theta + \frac{(n-1)\alpha}{2}\right)$$

$$\cos\theta + \cos(\theta+\alpha) + \cdots + \cos(\theta+(n-1)\alpha) = \frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\cos\left(\theta + \frac{(n-1)\alpha}{2}\right)$$

Important Trigonometric Values

15° AND 75°

$$\sin 15^\circ = \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$$

$$\cos 15^\circ = \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$$

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$$

$$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$$

18° AND 36°

$$\sin 18^\circ = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4}$$

$$\cos 36^\circ = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$$

Trigonometric Equations - General Solutions

$$\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha, \quad n \in \mathbb{Z}$$

$$\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha, \quad n \in \mathbb{Z}$$

$$\tan\theta = \tan\alpha \implies \theta = n\pi + \alpha, \quad n \in \mathbb{Z}$$

Inverse Trigonometric Functions - Domain & Range

$$\sin^{-1}x : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\cos^{-1}x : [-1,1] \to [0,\pi]$$

$$\tan^{-1} x : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$\cot^{-1} x : \mathbb{R} \to (0, \pi)$$

$$\sec^{-1} x : (-\infty, -1] \cup [1, \infty) \to [0, \pi], x \neq \frac{\pi}{2}$$

$$\csc^{-1} x : (-\infty, -1] \cup [1, \infty) \to [-\frac{\pi}{2}, \frac{\pi}{2}], x \neq 0$$

Inverse Trig - Complementary Relations

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad -1 \leq x \leq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in \mathbb{R}$$

$$\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}, \quad |x| \geq 1$$

Inverse Trig - Negative Arguments

$$\sin^{-1}(-x) = -\sin^{-1} x, \quad -1 \leq x \leq 1$$

$$\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in \mathbb{R}$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x, \quad -1 \leq x \leq 1$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x, \quad x \in \mathbb{R}$$

Inverse Trig - Addition Formulas

SINE ADDITION

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), \text{ if } x, y \geq 0, x^2 + y^2 \leq 1$$

COSINE ADDITION

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}),\ \text{if } x, y \geq 0$$

TANGENT ADDITION

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy},\ \text{if } xy < 1$$

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy},\ \text{if } x, y > 0, xy > 1$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Inverse Trig - Important Results

$$\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x,\ \text{if } |x| \leq \frac{1}{\sqrt{2}}$$

$$\cos^{-1}(2x^2 - 1) = 2\cos^{-1} x,\ \text{if } 0 \leq x \leq 1$$

$$\tan^{-1} \frac{2x}{1-x^2} = 2\tan^{-1} x,\ \text{if } |x| < 1$$

TRIPLE ADDITION

$$\text{If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi, \text{then } x + y + z = xyz$$

$$\text{If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}, \text{then } xy + yz + zx = 1$$

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$

Coordinate Geometry - Straight Lines

Distance and Section Formula

DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SECTION FORMULA (INTERNAL)

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

SECTION FORMULA (EXTERNAL)

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

MIDPOINT

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Special Points in Triangle

CENTROID

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

INCENTRE

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

where a, b, c are side lengths

EXCENTRE

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

AREA OF TRIANGLE

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Straight Line - Slope & Angles

SLOPE

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$$

ANGLE BETWEEN LINES

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

COLLINEARITY CONDITION

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

PARALLEL LINES

$$\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$$

PERPENDICULAR LINES

$$aa' + bb' = 0 \text{ or } m_1 m_2 = -1$$

Distance Formulas for Lines

POINT TO LINE

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

BETWEEN PARALLEL LINES

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

FOOT OF PERPENDICULAR

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

REFLECTION OF POINT

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

Angle Bisectors

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

$$undefined$$

> *Acute angle bisector: $aa' + bb' > 0$ (same sign), Obtuse: $aa' + bb' < 0$ (opposite sign)*

Concurrency of Lines

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Pair of Straight Lines Through Origin

GENERAL FORM

$$ax^2 + 2hxy + by^2 = 0$$

ANGLE BETWEEN LINES

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

> *Lines perpendicular if $a + b = 0$, coincident if $h^2 = ab$*

Circles

Standard Equations of Circle

GENERAL FORM

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre: $(-g, -f)$, *Radius:* $r = \sqrt{g^2 + f^2 - c}$

STANDARD FORM

$$(x-h)^2 + (y-k)^2 = r^2$$

Centre: (h, k), *Radius:* r

PARAMETRIC FORM

$$x = h + r\cos\theta, \quad y = k + r\sin\theta$$

INTERCEPTS ON AXES

$$x\text{-axis: } 2\sqrt{g^2 - c}, \quad y\text{-axis: } 2\sqrt{f^2 - c}$$

Tangent to Circle

SLOPE FORM

$$y = mx \pm a\sqrt{1 + m^2}$$

POINT FORM

$$xx_1 + yy_1 = a^2 \quad (\text{or } T = 0)$$

FOR GENERAL CIRCLE

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

PARAMETRIC FORM

$$x\cos\theta + y\sin\theta = a$$

LENGTH OF TANGENT

$$L = \sqrt{S_1} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

Pair of Tangents & Chord

PAIR OF TANGENTS

$$SS_1 = T^2$$

> *S: circle equation, S_1: value at point, T: chord of contact*

CHORD OF CONTACT

$$T = 0 : xx_1 + yy_1 = a^2$$

LENGTH OF CHORD OF CONTACT

$$\frac{2RL}{\sqrt{R^2 + L^2}}$$

> *R = radius, L = length of tangent*

AREA OF TRIANGLE

$$\frac{RL^3}{R^2 + L^2}$$

ANGLE BETWEEN TANGENTS

$$\tan\theta = \frac{2RL}{L^2 - R^2}$$

Important Circle Results

DIRECTOR CIRCLE

$$x^2 + y^2 = 2a^2 \text{ for } x^2 + y^2 = a^2$$

> *Locus of intersection of perpendicular tangents*

ORTHOGONALITY CONDITION

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

> *Two circles cut orthogonally*

RADICAL AXIS

$$S_1 - S_2 = 0 : 2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$$

FAMILY OF CIRCLES

$$S_1 + \lambda S_2 = 0 \text{ or } S + \lambda L = 0$$

Equation of Circle Circumscribing Triangle

$$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$$

> *Circle through point (x_1, y_1) and tangent points from it*

Conic Sections - Parabola, Ellipse & Hyperbola

Parabola - Standard Forms

STANDARD EQUATION

$$y^2 = 4ax$$

Vertex: $(0, 0)$, *Focus:* $(a, 0)$, *Directrix:* $x = -a$

PARAMETRIC FORM

$$x = at^2, \quad y = 2at$$

OTHER FORMS

$$y^2 = -4ax \text{ (left)}, \quad x^2 = 4ay \text{ (up)}, \quad x^2 = -4ay \text{ (down)}$$

FOCAL CHORD

$$t_1 t_2 = -1$$

For parametric points t_1, t_2

LENGTH OF LATUS RECTUM

$$4a$$

Parabola - Tangent & Normal

TANGENT (SLOPE)

$$y = mx + \frac{a}{m}$$

TANGENT (POINT)

$$yy_1 = 2a(x + x_1) \quad \text{or } T = 0$$

TANGENT (PARAMETRIC)

$$ty = x + at^2$$

NORMAL (PARAMETRIC)

$$y + tx = 2at + at^3$$

CHORD OF CONTACT

$$yy_1 = 2a(x + x_1)$$

Ellipse - Standard Forms

STANDARD EQUATION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

> *Foci:* $(\pm ae, 0)$, $e = \sqrt{1 - \frac{b^2}{a^2}}$

PARAMETRIC FORM

$$x = a\cos\theta, \quad y = b\sin\theta$$

ECCENTRICITY

$$e = \frac{\sqrt{a^2-b^2}}{a}, \quad b^2 = a^2(1-e^2), \quad (0

LENGTH OF LATUS RECTUM

$$\frac{2b^2}{a}$$

FOCAL DISTANCES

$$SP + S'P = 2a$$

> *Sum of focal distances is constant*

$$\text{Directrices: } x = \pm\frac{a}{e}$$

Ellipse - Tangent & Normal

TANGENT (SLOPE)

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

TANGENT (POINT)

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \text{or } T = 0$$

TANGENT (PARAMETRIC)

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

NORMAL (PARAMETRIC)

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

DIRECTOR CIRCLE

$$x^2 + y^2 = a^2 + b^2$$

Hyperbola - Standard Forms

STANDARD EQUATION

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

> *Foci:* $(\pm ae, 0)$, $e = \sqrt{1 + \frac{b^2}{a^2}}$

PARAMETRIC FORM

$$x = a\sec\theta, \quad y = b\tan\theta$$

ECCENTRICITY

$$e = \frac{\sqrt{a^2+b^2}}{a}, \quad b^2 = a^2(e^2-1), \quad (e>1)$$

LENGTH OF LATUS RECTUM

$$\frac{2b^2}{a}$$

ASYMPTOTES

$$y = \pm\frac{b}{a}x \text{ or } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

FOCAL DISTANCES

$$|SP - S'P| = 2a$$

> *Difference of focal distances is constant*

$$\text{Directrices: } x = \pm\frac{a}{e}$$

Hyperbola - Tangent & Rectangular

TANGENT (SLOPE)

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

TANGENT (POINT)

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \quad \text{or } T = 0$$

RECTANGULAR HYPERBOLA

$$xy = c^2, \quad e = \sqrt{2}$$

PARAMETRIC (RECTANGULAR)

$$x = ct, \quad y = \frac{c}{t}$$

TANGENT (RECTANGULAR)

$$\frac{x}{t} + yt = 2c$$

Limits and Continuity

Fundamental Limit Theorems

SUM RULE

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

PRODUCT RULE

$$\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

QUOTIENT RULE

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \quad \lim_{x \to a} g(x) \neq 0$$

SANDWICH THEOREM

$$\text{If } f(x) \leq g(x) \leq h(x) \text{ and } \lim f(x) = \lim h(x) = L, \text{ then } \lim g(x) = L$$

Standard Trigonometric Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$$

$$\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$$

Exponential & Logarithmic Limits

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a}$$

$$\lim_{x\to 0} (1+x)^{1/x} = e$$

$$\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x\to 0} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

L'Hôpital's Rule & Indeterminate Forms

L'HÔPITAL'S RULE

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

> *For $\frac{0}{0}$ or $\frac{\infty}{\infty}$ forms*

FORMS

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, 0^0, 1^\infty, \infty^0$$

> *Seven indeterminate forms*

Important Series Expansions

EXPONENTIAL

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

LOGARITHM

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad |x| < 1$$

SINE

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

COSINE

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

TANGENT

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots$$

BINOMIAL

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Differential Calculus

Basic Derivatives

$$\frac{d}{dx}(c) = 0$$

Constant

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Power rule

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Trigonometric Derivatives

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Inverse Trigonometric Derivatives

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Differentiation Rules

SUM/DIFFERENCE

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

PRODUCT RULE

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

CHAIN RULE

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

IMPLICIT

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

> *For $F(x, y) = 0$*

Applications of Derivatives

TANGENT EQUATION

$$y - y_1 = f'(x_1)(x - x_1)$$

NORMAL EQUATION

$$y - y_1 = -\frac{1}{f'(x_1)}(x - x_1)$$

MAXIMA/MINIMA

$$f'(x) = 0, \quad f''(x) < 0 \text{ (max)}, \quad f''(x) > 0 \text{ (min)}$$

POINT OF INFLECTION

$$f''(x) = 0 \text{ and } f'''(x) \neq 0$$

ROLLE'S THEOREM

$$f'(c) = 0 \text{ for some } c \in (a, b)$$

> *If $f(a) = f(b)$ and f continuous*

MEAN VALUE THEOREM

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ for some } c \in (a, b)$$

Integral Calculus

Basic Integration Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$$

Trigonometric Integration

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \tan x \, dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

Special Algebraic Integration

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \ln|x + \sqrt{x^2 + a^2}| + C$$

$$\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

Integration Techniques

BY PARTS

$$\int u\,dv = uv - \int v\,du$$

> *ILATE: Inverse, Log, Algebraic, Trig, Exponential*

SUBSTITUTION

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du, \quad u = g(x)$$

PARTIAL FRACTIONS

$$\frac{P(x)}{Q(x)} = \sum \frac{A_i}{(x-a_i)^k} + \sum \frac{B_i x + C_i}{(x^2+bx+c)^m}$$

Definite Integration Properties

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

> *Variable change*

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

> *Limits reversal*

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

> *Additivity*

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

> *King's property*

$$\int_0^{2a} f(x)\,dx = \int_0^a [f(x) + f(2a-x)]\,dx$$

$$\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x) = f(x) \\ 0 & \text{if } f(-x) = -f(x) \end{cases}$$

> *Even/Odd*

Area & Applications

AREA UNDER CURVE

$$A = \int_a^b |y|\, dx = \int_a^b |f(x)|\, dx$$

AREA BETWEEN CURVES

$$A = \int_a^b |f(x) - g(x)|\, dx$$

VOLUME OF REVOLUTION

$$V = \pi \int_a^b [f(x)]^2\, dx$$

> *About x-axis*

ARC LENGTH

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Quadratic Equations

Standard Form & Roots

STANDARD FORM

$$ax^2 + bx + c = 0, \quad a \neq 0$$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT

$$D = b^2 - 4ac$$

SUM OF ROOTS

$$\alpha + \beta = -\frac{b}{a}$$

PRODUCT OF ROOTS

$$\alpha\beta = \frac{c}{a}$$

EQUATION FROM ROOTS

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Nature of Roots

$$D > 0 \implies \text{Real and distinct roots}$$

$$D = 0 \implies \text{Real and equal roots}$$

$$D < 0 \implies \text{Complex roots}$$

$$D \geq 0 \text{ and perfect square } \implies \text{Rational roots}$$

> *When a, b, c are rational*

COMMON ROOT CONDITION

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

> *For both roots common*

Location of Roots

BOTH ROOTS > K

$$D \geq 0, \quad -\frac{b}{2a} > k, \quad f(k) > 0$$

BOTH ROOTS < K

$$D \geq 0, \quad -\frac{b}{2a} < k, \quad f(k) > 0$$

ROOTS ON OPPOSITE SIDES OF K

$$f(k) < 0$$

EXACTLY ONE ROOT IN (K_1, K_2)

$$f(k_1) \cdot f(k_2) < 0$$

Sequences & Series

Arithmetic Progression (AP)

GENERAL TERM

$$a_n = a + (n-1)d$$

SUM OF N TERMS

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

> l = *last term*

ARITHMETIC MEAN

$$AM = \frac{a+b}{2}$$

N AMS BETWEEN A & B

$$A_k = a + k \cdot \frac{b-a}{n+1}, \quad k = 1, 2, \ldots, n$$

Geometric Progression (GP)

GENERAL TERM

$$a_n = ar^{n-1}$$

SUM OF N TERMS

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

SUM TO INFINITY

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1$$

GEOMETRIC MEAN

$$GM = \sqrt{ab}$$

N GMS BETWEEN A & B

$$G_k = a \cdot r^k, \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Harmonic Progression (HP) & Means

HP GENERAL TERM

$$\frac{1}{a_n} \text{ is in AP}$$

HARMONIC MEAN

$$HM = \frac{2ab}{a+b}$$

AM-GM-HM RELATION

$$AM \geq GM \geq HM$$

> *Equality when $a = b$*

$$GM^2 = AM \times HM$$

Special Series Sums

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{k=1}^{n} (2k-1) = n^2$$

> *Sum of first n odd numbers*

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = H_n$$

> *Harmonic series (no closed form)*

Binomial Theorem

Binomial Expansion

GENERAL EXPANSION

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

BINOMIAL COEFFICIENT

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

GENERAL TERM

$$T_{r+1} = \binom{n}{r} x^{n-r} y^r$$

> $(r+1)^{th}$ *term*

MIDDLE TERM(S)

$$\text{If } n \text{ even: } T_{\frac{n}{2}+1} \quad \text{If } n \text{ odd: } T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$$

TOTAL TERMS

$$n+1 \text{ terms in expansion}$$

Binomial Properties

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$\sum_{r=0}^{n} \binom{n}{r} = 2^n$$

> *Sum of binomial coefficients*

$$\sum_{r=0}^{n} (-1)^r \binom{n}{r} = 0$$

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$$

Greatest Term & Coefficient

GREATEST TERM

$$T_{r+1} \text{ where } r = \left\lfloor \frac{(n+1)|y|}{|x| + |y|} \right\rfloor$$

GREATEST COEFFICIENT

$$\binom{n}{\frac{n}{2}} \text{ if } n \text{ even}, \binom{n}{\frac{n\pm1}{2}} \text{ if } n \text{ odd}$$

Permutations & Combinations

Fundamental Principles

FACTORIAL

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1, \quad 0! = 1$$

ADDITION PRINCIPLE

$$\text{If task A in } m \text{ ways, task B in } n \text{ ways (mutually exclusive)} \implies m + n \text{ ways}$$

MULTIPLICATION PRINCIPLE

$$\text{If task A in } m \text{ ways, task B in } n \text{ ways (independent)} \implies m \times n \text{ ways}$$

Permutations

NPR

$${}^nP_r = \frac{n!}{(n-r)!}$$

> *Arrangements of r from n*

ALL ARRANGEMENTS

$${}^nP_n = n!$$

CIRCULAR PERMUTATION

$$(n-1)!$$

> *Arrangements in a circle*

REPETITION ALLOWED

$$n^r$$

> *r selections from n with repetition*

IDENTICAL OBJECTS

$$\frac{n!}{p!\, q!\, r!}$$

> *p, q, r identical of each type*

Combinations

NCR

$$^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

> *Selections of r from n*

$$^nC_r = {^nC_{n-r}}$$

$$^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$$

TOTAL SELECTIONS

$$2^n - 1$$

> *From n objects (excluding empty)*

DERANGEMENTS

$$D_n = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!} \approx \frac{n!}{e}$$

> *No object in original position*

Distribution Problems

DIVIDE N DISTINCT IN GROUPS

$$\frac{n!}{p!\,q!\,r!} \times \frac{1}{k!}$$

> *k groups identical*

NON-NEGATIVE INTEGER SOLUTIONS

$$^{n+r-1}C_{r-1}$$

> $x_1 + x_2 + \cdots + x_r = n$

Probability

Basic Probability

CLASSICAL DEFINITION

$$P(E) = \frac{n(E)}{n(S)}$$

For equally likely outcomes

$$0 \leq P(E) \leq 1$$

$$P(S) = 1, \quad P(\phi) = 0$$

COMPLEMENT

$$P(E') = 1 - P(E)$$

ADDITION RULE

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

FOR MUTUALLY EXCLUSIVE

$$P(A \cup B) = P(A) + P(B)$$

When $A \cap B = \phi$

Conditional Probability

DEFINITION

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

MULTIPLICATION THEOREM

$$P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

INDEPENDENT EVENTS

$$P(A \cap B) = P(A) \cdot P(B)$$

> *Also $P(A|B) = P(A)$*

BAYES' THEOREM

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum P(E_j) \cdot P(A|E_j)}$$

Random Variables & Distributions

EXPECTED VALUE

$$E(X) = \sum x_i \cdot P(x_i)$$

> *Mean or expectation*

VARIANCE

$$Var(X) = E(X^2) - [E(X)]^2$$

BINOMIAL DISTRIBUTION

$$P(X = r) = {}^nC_r \, p^r \, q^{n-r}, \quad q = 1 - p$$

> *n trials, r successes*

$$E(X) = np, \quad Var(X) = npq$$

> *For Binomial*

STANDARD DEVIATION

$$\sigma = \sqrt{Var(X)}$$

Complex Numbers

Basic Forms & Operations

STANDARD FORM

$$z = x + iy, \quad i = \sqrt{-1}, \quad i^2 = -1$$

CONJUGATE

$$\bar{z} = x - iy$$

MODULUS

$$|z| = \sqrt{x^2 + y^2}$$

ARGUMENT

$$\arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$$

POLAR FORM

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

> $r = |z|, \theta = \arg(z)$

Properties & Identities

$$z \cdot \bar{z} = |z|^2 = x^2 + y^2$$

$$|z_1 z_2| = |z_1| \cdot |z_2|$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

TRIANGLE INEQUALITY

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

De Moivre's Theorem & Roots

DE MOIVRE'S

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

EULER'S FORMULA

$$e^{i\theta} = \cos\theta + i\sin\theta$$

NTH ROOTS OF UNITY

$$z^n = 1 \implies z = e^{2\pi ik/n}, \quad k = 0, 1, 2, \ldots, n-1$$

$$1 + \omega + \omega^2 + \cdots + \omega^{n-1} = 0$$

> *Sum of n^{th} roots of unity*

CUBE ROOTS OF UNITY

$$\omega = e^{2\pi i/3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad 1 + \omega + \omega^2 = 0$$

Vectors

Vector Basics

POSITION VECTOR

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

MAGNITUDE

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

UNIT VECTOR

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

SECTION FORMULA

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

> *Divides AB in ratio $m : n$*

DIRECTION COSINES

$$\cos\alpha = \frac{l}{r}, \cos\beta = \frac{m}{r}, \cos\gamma = \frac{n}{r}, \quad l^2 + m^2 + n^2 = r^2$$

Dot Product (Scalar Product)

DEFINITION

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

COMPONENT FORM

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

> *Commutative*

PERPENDICULAR VECTORS

$$\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$$

PROJECTION

$$\mathrm{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Cross Product (Vector Product)

DEFINITION

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

> *$\hat{n}$ perpendicular to both*

COMPONENT FORM

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

> *Anti-commutative*

PARALLEL VECTORS

$$\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}$$

AREA OF PARALLELOGRAM

$$A = |\vec{a} \times \vec{b}|$$

AREA OF TRIANGLE

$$A = \frac{1}{2}|\vec{a} \times \vec{b}|$$

Scalar Triple Product

DEFINITION

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

COPLANARITY

$$[\vec{a}\,\vec{b}\,\vec{c}] = 0 \iff \text{vectors coplanar}$$

VOLUME OF PARALLELEPIPED

$$V = |[\vec{a}\,\vec{b}\,\vec{c}]|$$

VOLUME OF TETRAHEDRON

$$V = \frac{1}{6}|[\vec{a}\,\vec{b}\,\vec{c}]|$$

Three Dimensional Geometry

Line in 3D

VECTOR FORM

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$\vec{a}$ point, $\vec{b}$ direction

CARTESIAN FORM

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Direction ratios: l, m, n

TWO POINT FORM

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

ANGLE BETWEEN LINES

$$\cos\theta = \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|} = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

Plane in 3D

VECTOR FORM

$$\vec{r} \cdot \vec{n} = d$$

$\vec{n}$ normal vector

CARTESIAN FORM

$$Ax + By + Cz + D = 0$$

Normal: (A, B, C)

INTERCEPT FORM

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

DISTANCE FROM POINT

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

ANGLE BETWEEN PLANES

$$\cos\theta = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

Sphere

STANDARD FORM

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

> *Centre:* (a, b, c)*, Radius:* r

GENERAL FORM

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

> *Centre:* $(-u, -v, -w)$

DIAMETER FORM

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$$

Skew Lines & Distance

SHORTEST DISTANCE

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

> *Between skew lines*

DISTANCE (CARTESIAN)

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}}$$

Solution of Triangles

Fundamental Rules

SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

> *R = circumradius*

COSINE RULE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

PROJECTION FORMULA

$$a = b\cos C + c\cos B, \quad b = c\cos A + a\cos C, \quad c = a\cos B + b\cos A$$

Napier's Analogy (Tangent Rule)

$$\tan\frac{B - C}{2} = \frac{b - c}{b + c}\cot\frac{A}{2}$$

$$\tan\frac{C - A}{2} = \frac{c - a}{c + a}\cot\frac{B}{2}$$

$$\tan\frac{A - B}{2} = \frac{a - b}{a + b}\cot\frac{C}{2}$$

Half Angle Formulas

SINE

$$\sin\frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}, \quad s = \frac{a + b + c}{2}$$

COSINE

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

TANGENT

$$\tan\frac{A}{2} = \frac{\Delta}{s(s-a)} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

> *Δ = area*

$$\sin A = \frac{2\Delta}{bc}, \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

> *Heron's formula*

Area of Triangle

BASIC FORMS

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$$

HERON'S FORMULA

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$$

USING R

$$\Delta = \frac{abc}{4R}$$

Radii of Circles

CIRCUMRADIUS

$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}$$

INRADIUS

$$r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

EXRADII

$$r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2}, \quad r_2 = \frac{\Delta}{s-b}, \quad r_3 = \frac{\Delta}{s-c}$$

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

m-n Rule & Special Lengths

M-N RULE

$$(m + n) \cot \theta = m \cot B - n \cot C$$

> *If BD:DC = m:n*

MEDIAN LENGTH

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

ALTITUDE LENGTH

$$h_a = b \sin C = c \sin B = \frac{2\Delta}{a}$$

ANGLE BISECTOR

$$t_a = \frac{2bc \cos(A/2)}{b + c}$$

Statistics

Measures of Central Tendency

MEAN (UNGROUPED)

$$\bar{x} = \frac{\sum x_i}{n}$$

MEAN (GROUPED)

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{N}$$

MEDIAN

$$M = \begin{cases} x_{\frac{n+1}{2}} & \text{if } n \text{ odd} \\ \frac{x_{n/2} + x_{(n/2)+1}}{2} & \text{if } n \text{ even} \end{cases}$$

> *For sorted data*

MODE

$$\text{Value with maximum frequency}$$

Relationship Between Measures

SYMMETRIC DISTRIBUTION

$$\text{Mean} = \text{Median} = \text{Mode}$$

SKEWED DISTRIBUTION

$$\text{Mode} = 3\text{Median} - 2\text{Mean}$$

Measures of Dispersion

RANGE

$$R = \text{Largest value} - \text{Smallest value}$$

MEAN DEVIATION

$$MD = \frac{\sum |x_i - \bar{x}|}{n} \text{ or } \frac{\sum f_i |x_i - \bar{x}|}{N}$$

VARIANCE

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{\sum x_i^2}{n} - \bar{x}^2$$

STANDARD DEVIATION

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

COEFFICIENT OF VARIATION

$$CV = \frac{\sigma}{\bar{x}} \times 100\%$$

> *Relative measure*

Properties of Variance

$$\text{Var}(x_i + a) = \text{Var}(x_i)$$

> *Translation doesn't affect variance*

$$\text{Var}(ax_i) = a^2 \text{Var}(x_i)$$

> *Scaling by a*

$$\text{Var}(ax_i + b) = a^2 \text{Var}(x_i)$$

Mathematical Reasoning

Logical Connectives

NEGATION

$$\sim p \text{ (NOT p)}$$

CONJUNCTION

$$p \wedge q \text{ (p AND q)}$$

True when both true

DISJUNCTION

$$p \vee q \text{ (p OR q)}$$

True when at least one true

IMPLICATION

$$p \Rightarrow q \text{ (if p then q)}$$

False only when p true, q false

BICONDITIONAL

$$p \Leftrightarrow q \text{ (p if and only if q)}$$

True when both same

Special Statements

TAUTOLOGY

$$p \vee \sim p$$

Always true (e.g., p OR NOT p)

CONTRADICTION

$$p \wedge \sim p$$

Always false (e.g., p AND NOT p)

CONTRAPOSITIVE

$$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$$

> *Logically equivalent*

CONVERSE

$$q \Rightarrow p$$

> *Not necessarily equivalent to* $p \Rightarrow q$

INVERSE

$$\sim p \Rightarrow \sim q$$

> *Not necessarily equivalent*

Quantifiers

UNIVERSAL

$$\forall x \in S, P(x) \text{ (For all x in S, P(x) is true)}$$

EXISTENTIAL

$$\exists x \in S, P(x) \text{ (There exists x in S such that P(x) is true)}$$

NEGATION OF UNIVERSAL

$$\sim (\forall x, P(x)) \equiv \exists x, \sim P(x)$$

NEGATION OF EXISTENTIAL

$$\sim (\exists x, P(x)) \equiv \forall x, \sim P(x)$$

Sets & Relations

Set Operations & Laws

UNION

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

INTERSECTION

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

DIFFERENCE

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

COMPLEMENT

$$A' = U - A = \{x : x \in U \text{ and } x \notin A\}$$

DE MORGAN'S LAWS

$$(A \cup B)' = A' \cap B', \quad (A \cap B)' = A' \cup B'$$

Cardinality Formulas

TWO SETS

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

DIFFERENCE

$$n(A - B) = n(A) - n(A \cap B)$$

THREE SETS

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

POWER SET

$$n(P(A)) = 2^{n(A)}$$

> *Number of subsets*

Types of Relations

REFLEXIVE

$$(a, a) \in R \text{ for all } a \in A$$

> *Every element related to itself*

SYMMETRIC

$$(a, b) \in R \Rightarrow (b, a) \in R$$

TRANSITIVE

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$$

EQUIVALENCE

$$\text{Reflexive} + \text{Symmetric} + \text{Transitive}$$

IDENTITY RELATION

$$I_A = \{(a, a) : a \in A\}$$

Functions

ONE-ONE (INJECTIVE)

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

ONTO (SURJECTIVE)

$$\text{Range} = \text{Codomain}$$

BIJECTIVE

$$\text{One-One and Onto}$$

> *Has inverse*

INVERSE FUNCTION

$$f^{-1}(f(x)) = x \text{ and } f(f^{-1}(y)) = y$$